Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 19: Financial Maths I

19.1 Learning Intentions

After this week's lesson you will be able to;

- Use tables to represent patterns.
- Outline an Arithmetic (Linear) Sequence.
- Outline an Arithmetic (Linear) Series.
- Describe the general term for a linear sequence.
- Work with Quadratic and Cubic sequences in the same way.
- Describe an exponential sequence.
- Outline what a geometric sequence is. Look at limits and infinite series.

19.2 Specification

3.1 Number systems (continued)	– appreciate that processes can generate sequences of numbers or objects – investigate patterns among these sequences – use patterns to continue the sequence – generalise and explain patterns and relationships in algebraic form – recognise whether a sequence is arithmetic, geometric or neither – find the sum to n terms of an arithmetic series	– verify and justify formulae from number patterns – investigate geometric sequences and series

19.3 Chief Examiner's Report

The highest mean mark was 88% in Question 3 (functions). Candidates also performed particularly well on Question 1 (sequences), Question 2 (algebra), and Question 6 (financial applications of sequences).

19.4 Arithmetic Sequences

A sequence is a list number separated by commas. There are many different types of sequences but the first one we will look at is arithmetic or linear.

3 5 7 9 11

Term Number	T_1	T_2	T_3	T_4	T_5
Term Value					
General Form					

In order to establish any term in given sequence we can use this:

$$\textbf{General Term } (T_n) = a + (n-1)d$$

19.5 Arithmetic Series

5 7 9 11

The sum of these five tersm=

We can rewrite that as S_5=35

To calculate the sum for any numbe of terms, such as the first n terms we can use the below formula which can also be found on pg22 of the Formula and Table book:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Use this formula now to verify that S_5=35.

a=

n=

d=

S_s

For a particular sequence $S_n = 2n - 5n^2$, Show that $T_n = 7 - 10_n$.

Use the below space to take down the solution:

19.6 Quadratic Sequence

This is the first of 3 non-linear sequences we will look at on our course. In order to establish if a sequence is quadratic or not we examine the differences:

3 5 8 12 17

We notice that for a quadratic equation the ____________is common. However, we can also still use the idea of predicting future terms with the concept Of Tn. HOwever, it differs slightly for quadratic sequences.

$$T_n = an^2 + bn + c$$

Also, we need to remember that a is not the first term for a quadratic sequence, instead it's:

$$2a = 2^{nd}\ difference$$

Use the space below to copy down the solution to finding T for the above sequence:

19.7 Cubic Sequence

The same pattern follows on for determining if a sequence is a cubic sequence or not.

2 10 30 68 130

$$T_n = an^3 + bn^2 + cn + d$$

19.8 Exponential Sequence

This is any from sequence that appears to increase/decrease based on the multiplication of a term to produce the following term. For example:

8 4 2 1 0.5

Term Number	T_1	T_2	T_3	T_4	T_5
Term Value	8	4	2	1	0.5
General Form					

19.9 Geometric Sequence

These types of sequences are those that are made by multiplying the first term of the sequence by a certain number. Can you spot the number in the below sequence?

5 10 20 40

We call this number the **common ratio** or r. Each of the terms can be found by multiplying by the common ratio to a certain power, this yields the formula:

$$T_n = ar^{n-1}$$

We can also calculate the sum of terms in such a sequence similar to that of a linear sequence:

$$S_n = \frac{a(1-r^n)}{1-r}$$

All of these concepts can be used to solve practical problems, just like the one below:

The population of a town is pre.Gently 38,300 and there is a population growth of 1.2 to annuall y. Find the number of years it takes for the population i‹› reach 43,158

19.10 Limits

A limit is can be descrribed as the unreachable destination of a sequence.

8 4 2 1 0.5 →

This si a **Convergent** ssequence as there is a limit.

3 5 7 9 11 →

This is a **Divergent** sequence as there is **NO** limit
When evaluating limits, we divide by the highest power of n that is present.

$$Evaluate\ \lim_{n\to\infty} \frac{2n^2 - 15n - 1}{3n^2 - 3n + 2}$$

Sometimes you may need to use of the laws of limits to help siplify an expression before dividing by n^2.

Laws ofLimits:

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) \qquad \text{Sum Law}$$

$$\lim_{x\to a}[f(x) \times g(x)] = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x) \qquad \text{Product Law}$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \qquad \text{Quotient Law}$$

$$\lim_{x\to a}\sqrt{f(x)} = \sqrt{\lim_{x\to a} f(x)} \qquad \text{Root Law}$$

19.11 Infinite Series

This is the case when we have a series that does not appear to ever end such as:

$$\sum_{n=1}^{\infty} a_n =$$

But we can use limits to help us with solving such a problem. If we are looking at the sequence and establish what r is, we can decide if this approach will work or not. If |r | < 1 t hen we can use this sum formula:

$$S_\infty = \frac{a}{1-r}$$

This shows us that the sequence converges. So, we can investigate if the common ratio is less than 1 to be able to tell if a sequence converges.

19.12 Recap of the Learning Intentions

After this week's lesson you will be able to; Use tables to represent patterns.

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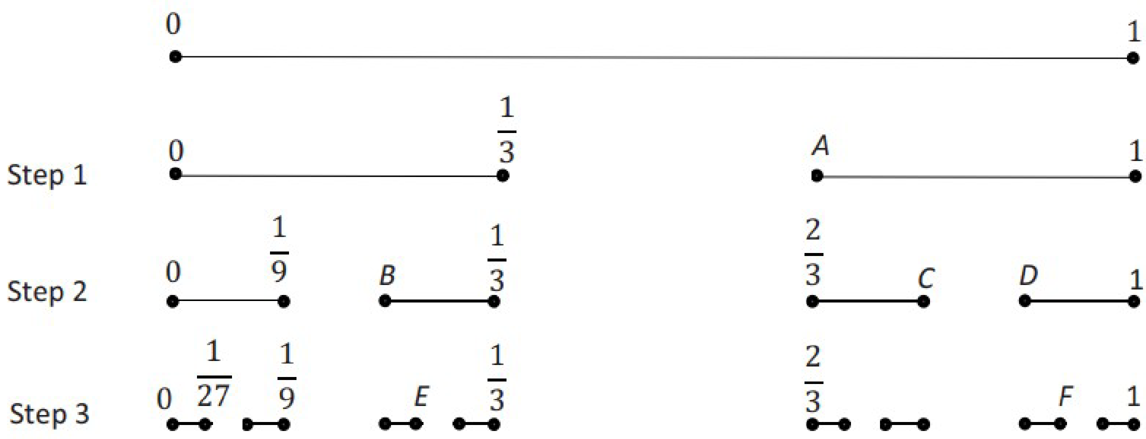
19.13 Homework Task

The closed line segment [0, 1] is shown below. The first three steps in the construction of the Cantor Set are also shown:

- Step 1 removes the open middle third of the line segment [0, 1] leaving two **closed line segments** (i.e. the end points of the segments remain in the Cantor Set)

- Step 2 removes the middle third of the two remaining segments leaving four closed line segments
- Step 3 removes the middle third of the four remaining segments leaving eight closed line segments.

The process continues **indefinitely**. The set of points in the line segment [0, 1] that are not removed during the process is the Cantor Set.



(a) **(i)** Complete the table below to show the length of the line segment(s) removed **at each step** for the first 5 steps. Give your answers as fractions.

Step	Step1	Step 2	Step 3	Step 4	Step 5
Length Removed	$\frac{1}{3}$	$\frac{2}{9}$			

(ii) Find the **total** length of all of the line segments removed from the initial line segment of length 1 unit, after a finite number (n) of steps in the process.
Give your answer in terms of n.

(iii) Find the total length removed, from the initial line segment, after an infinite number of steps of the process.

(b) **(i)** Complete the table below to identify the end-points labelled in the diagram. Give your answers as **fractions.**

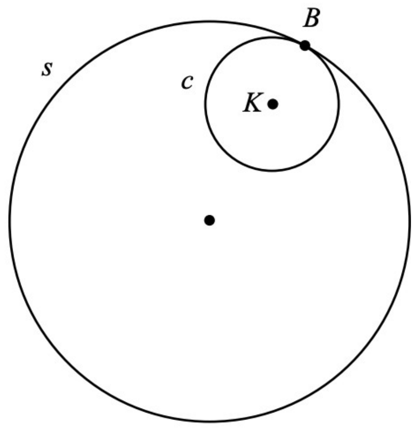
Label	A	B	C	D	E	F
End-point						

(ii) Give a reason why $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\frac{1}{81}$ is a point in the *Cantor Set*.

(iii) The limit of the series $\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\cdots$ is a point in the *Cantor Set*. Find this point.

19.14 Solutions to 18.11

Two circles s and c touch internally at B, as shown.



(a) The equation of the circle s is

$(x-1)^2+(y+6)^2=360.$

Write down the co-ordinates of the centre of s.

Centre: (1, —6)

Write down the radius of s in the form $a\sqrt{10}$, where $a \in \mathbb{N}$.

Radius: $\overline{360}$

(b) **(i)** The point K is the centre of circle c.
The radius of c is one-third the radius of s.
The co-ordinates of B are (7, 12).
Find the co-ordinates of K.

centre o f circle s to Point *B is a* translation *o f x* up 6 and y up 18.

As we are only trnveflinp 2/3 *o f this* trnnslntion *we:*

$\frac{2}{3}$ (6) *is the x translation and* $\frac{2}{3}$ (18) *is the* y *translation; K* —— (5, 6)

(ii) Find the equation of c.

$$(x-5)^2+(y-6)^2=\left(2\sqrt{10}\right)^2$$

(c) Find the equation of the common tangent at B.
Give your answer in the form $ax+by+c=0$, where $a, b, c \in \mathbb{Z}$.

$$Slope\ of\ AB=\frac{12+6}{7-1}=3$$

$$As\ tangent\ is\ \perp\ to\ AB\ it\ has\ a\ slope\ of\ -\frac{1}{3}$$

$$y-12=-\frac{1}{3}(x-7)$$

$$x+3y-43=0$$